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FREE SURFACE FLOWS WITHOUT WAVES

BY

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In this paper we explore, within the framework of first order theory, the existence of wave-free flows past submerged bodies in both two and three dimensions. This is done through consideration and mutual cancellation ofthe wave fields due to singularities and singularity distributions which can be interpreted in terms of body volume and vertical force distributions. The results may be of practical consequence for ship wave-making and also in connection with cavity flows beneath a free surface.

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1. Introduction and Summary. The motion of bodies under a free surface in the presence of gravity usually leads to waves on the surface which propagate energy away from the body. The latter experiences a corresponding resistance to motion, which is of considerable practical importance for ships and other marine vehicles.

In this paper we explore, within the framework of first order theory, the existence of wave-free flows past submerged bodies in both two and three dimensions. This is done through consideration and mutual cancellation of the wave fields due to singularities (Green's Functions) and singularity distributions, which can be interpreted in terms of body volume and vertical force (lift) distributions. The results may be of practical consequence for ship wave-making and also in connection with cavity flows beneath a free surface.

We show that:

1) In the absence of any dynamic lift, resistance-free bodies can exist only in two dimensions only for Froude numbers: $F_R = U_o^2/gL < 1/2\pi$. However, under that condition an infinite number of different closed bodies can be found which have no wave resistance. The non-existence of resistance-free, ship-like bodies comprised solely of horizontal doublets (as in Michell's theory) had

earlier been noted. (U_o, speed of body; L, characteristic body length; g, acceleration of gravity.)

2) When dynamic lift is allowed, resistance-free bodies exist at all Froude numbers and in infinite number both in two and three dimensions.

3) Two general types of wave-free singularity systems exist in two and three dimensions, involving linear combinations of singularity distributions representing volume (horizontal doublet) and lift (vertical doublet) separately. In the first of these (type I), the vertical doublet strength required to cancel the waves due to a horizontal doublet decreases as F_R^{-1}, and involves a net dynamic lift acting on the body. In type II, the ratio of vertical to horizontal doublet strengths increases as F_R, and the net dynamic lift can be null.

4) Within approximations valid when the cylinder is not too close to the free surface, the planar flow past a circular cylinder beneath a free surface is wave free of type I provided a circulation exists around the cylinder (as due to its rotation) so as to provide a downward directed force equal to twice the buoyancy acting upward on the cylinder.

5) Within linearizing approximations, the flow past a slender submerged body in two and three dimensions is wave free provided that the body is cambered so that at each vertical section dynamic

lift is generated exactly to annul the upward buoyancy force due to gravity.

6) In a cavity flow past a forebody, the shape of the long trailing constant pressure cavity must adjust itself so that the buoyancy force at each section is exactly balanced by dynamic lift. As a result, cavities created beneath a free surface in either two or three dimensions are not of themselves a source of trailing waves or of wave resistance, despite their substantial volume.

Finally, in this paper, the practical implications of wave-free singularity systems is discussed with particular emphasis on the role of vertical force distribution in minimizing the wave resistance of ships, an important aspect generally neglected previously in both theory and practice.

2. Approximations. The energy loss associated with the wave resistance of a body is found as the sum of terms including the wave energy radiated far away from the body plus the dissipation which occurs in the vicinity of a breaking surface. We neglect breaking in all that follows, and viscosity. Therefore, a velocity potential exists

The theoretical treatment of wave-making even under these circumstances is complicated by three well-known difficulties: i) the shape of the free surface is not known a priori; ii) the boundary conditions on the free surface are of non-linear nature, and iii) the hulls of most ships are of such a shape that difficulties ensue in satisfying boundary conditions upon them. These difficulties are so great, that not a single example of

an exact flow past a body moving under a free surface with gravity is available.

The difficulties are avoided at once if: i) so-called linearized boundary conditions are employed, and ii) singularity systems rather than hull forms are considered from the outset. Of course, the utility of singularity systems arises through superposition, and therefore depends upon the validity of linearization. The question immediately arises whether linearization is justified.

Linearized wave theory has been highly successful in predicting the form and allowing general interpretations of the complex wave patterns produced by moving disturbances. It also provides, as in the form of Michell's theory, a general understanding of the relation between ship wave resistance and the ship's hull form. It has not, however, yet provided predictions useful in ship design. Nor are the general convergence properties of linearized expansions understood or, therefore, the extent of their usefulness. Under these circumstances it seems wise especially to utilize linearized theory for gaining further understanding of wave-making phenomena and hopefully to reveal important principles which may be applied, in design or in interpretation of model tests, with the help of common sense and experience. That is the spirit of the present work.

Beyond these general remarks, what can be said about the relationship between the linearized and "real" wave resistance? It should be expected that the simplest form for the second order resistance will be:

$$R_2[F,\delta,h] = R_1[F+O(\delta);\delta;h + O(\delta)] + A\delta^4 \qquad (2.1)$$

where δ is proportional to the singularity strength and where $R_1 = O(\delta^2)$. Notice that, beside the addition of the expected higher order contribution, the first order resistance is subject to a re-interpretation which manifests itself in a straining of the independent parameters, F and h, arising through the inclusion of first order terms in the second order boundary conditions. On account of this effect, the second order correction may become particularly large where the resistance is changing rapidly with F and or h, but will be minimized at minima of the resistance curve. In particular, a linearized flow of zero wave resistance ($R_1 = 0$) may be interpreted as representing a "real" flow with a wave resistance of $O(\delta^4)$. These facts give special significance to linearized theory applied to flows with little or no wave resistance.

3. <u>The Free Surface Condition</u>. Upon the free surface:

$$q^2/2 + gy = 0 \tag{3.1}$$

where q is the flow speed and y the surface elevation. The first order expression of this condition is that, upon the <u>undisturbed</u> free surface ($y = 0$):

$$\varphi_{xx} + \frac{g}{U_o^2}\,\varphi_y = 0 \tag{3.2}$$

where φ is the potential, in either two or three dimensions, associated with the disturbance, g is the acceleration of gravity, and U_o the relative speed (in the x direction) at infinity.

In two dimensions, this condition may also be expressed in terms of the stream function, $\psi = -U_o y$, and becomes, on the x-axis:

$$\psi_y - \frac{g}{U_o^2}\psi = 0 \tag{3.3}$$

or, in terms of the complex potential $\Psi = \varphi + i\psi$,

$$R\left[\Psi' + \frac{ig}{U_o^2}\Psi\right] = 0 \quad \text{on the real axis.} \tag{3.4}$$

4. Singularity Systems. To an approximation, submerged singularities may be given definite physical meaning in terms of the shape of bodies which they represent and the forces acting on those bodies. In slender body theory, sources and horizontal doublets represent body volume and volume changes, while vertical doublets and vortices represent vertical force generated by flow. Higher order singularities represent shape alteration without change of lift or volume. Image singularities, imagined to exist outside the physical plane are invoked to satisfy boundary conditions on the free surface. Those which trail to the rear in the image plane above the free surface give rise to the radiated wave field itself. The problem of the elimination of wave resistance may thus be interpreted in terms of elimination of the trailing image singularities.

5. Image Systems Planar Flows. As an example which is readily generalized, let us consider the flow field associated with a source of strength m located at a depth, y_o, beneath the undeflected free surface. This flow is well known, but we shall derive it utilizing analytic function methods and so that an immediate interpretation may be made of it in terms of the image singularity system. Let the flow we seek be $\Psi = \Psi_o + \Psi_1$, where,

$$\Psi_o = \Psi_m(z+z_o) + \Psi_m(z-z_o), \quad z_o = iy_o \tag{5.1}$$

so that,

$$I[\Psi_o] = 0 \quad \text{and} \quad R[\Psi_o'] = 2R[\Psi_m'(z-z_o)] \quad \text{on the real axis} \tag{5.2}$$

and, in view of (3.4), and (5.2) above,

$$R[\Psi_1' + \frac{ig}{U_o^2}\Psi_1 + 2\Psi_m'(z-z_o)] = 0 \quad \text{on the real axis} \tag{5.3}$$

Since both Ψ_1 and $\Psi_m(z-z_o)$ are regular beneath the half plane $y = 0$, the entire function within the brackets must be identically zero, i.e.,

$$\frac{d\Psi_1}{dz} + \frac{ig}{U_o^2}\Psi_1 = -2\frac{d\Psi_m(z-z_o)}{dz} \tag{5.4}$$

and since $\Psi_1 = \Psi_1(z-z_o)$, this may also be written:

$$-\frac{d\Psi_1}{dz_o} + \frac{ig}{U_o^2}\Psi_1 = -2\frac{d\Psi_m(z-z_o)}{dz} \tag{5.5}$$

This differential equation has the solution:

$$\Psi_1 = -2\int_{z_o}^{\infty}\frac{d\Psi_m(z-z'')}{dz}e^{ig/U_o^2(z_o-z'')}dz'' \tag{5.6}$$

or, alternatively,

$$\Psi_1 = -2\Psi_m(z-z_o) + \frac{2ig}{U_o^2}\int_{z_o}^{\infty}\Psi_m(z-z'')e^{ig/U_o^2(z_o-z'')}dz'' \tag{5.7}$$

where we have purposely avoided waves at infinity upstream. The

equivalent of these results was obtained earlier by Havelock [1927] in a different way.

The potentials corresponding to (5.6) and (5.7) are:

$$\varphi_o = \varphi_m(x;y+y_o)+\varphi_m(x;y-y_o)-2\int_0^\infty\left[\frac{\partial\varphi_m}{\partial x}(x-x'';y-y_o;z)\cos\frac{g}{U_o^2}(x'') - \frac{\partial\varphi_m}{\partial y}(x-x'';y-y_o;z)\sin\frac{g}{U_o^2}(x'')\right]dx'' \tag{5.8}$$

and

$$\varphi_o = \varphi_m(x;y+y_o)-\varphi_m(x;y-y_o)+\frac{2g}{U_o^2}\int_0^\infty\left[\varphi_m(x-x'';y-y_o;z)\cos\frac{g}{U_o^2}(x'') + \psi_m(x-x'';y-y_o;z)\sin\frac{g}{U_o^2}(x'')\right]dx'' \tag{5.9}$$

where ψ_m represents a point vortex. The integrals in the expressions above clearly correspond to regular periodic singularity distributions, of dipoles in (5.8), and sources and vortices in (5.9).

Quite clearly these oscillating singularities must lead to downstream oscillations in the shape of the free surface. The wavelength of these oscillations is:

$$\lambda = 2\pi U_o^2/g \tag{5.10}$$

The integral in (5.6) may also be evaluated by contour integration. The appropriate path, avoiding waves at infinity upstream lies along $z'' = x + iy_o$ for $x > 0$ and $z'' = iy$ for $y < y_o$. The result is:

$$
Y_1 = \begin{cases} -2mi e^{-g/U_o^2[ix+(y_o-y)]} & x > 0 \\ 0 & x < 0 \end{cases}
$$

$$
- \left\{ \int_{-\infty}^{y_o} \frac{(y-y'')e^{-g/U_o^2(y_o-y'')}}{[x^2 + (y-y'')^2]} dy'' + ix \int_{-\infty}^{y_o} \frac{e^{-g/U_o^2(y_o-y'')}}{[x^2 + (y-y'')^2]} dy'' \right\} \tag{5.11}
$$

The term in the first pair of brackets represents a surface wave starting at $x = 0$ and running downstream. The second term is a local disturbance vanishing for large $|x|$.

The image systems and velocity fields due to general planar singularity distributions are readily derived from the above results by differentiation and superposition.

6. Source Distributions Critical Spacing. The trailing image singularities represented by the infinite integrals which occur in (5.8) and (5.?) must be responsible for the wave resistance, since the disturbances due to singularities restricted to the finite plane will decay too rapidly to allow for the convection of energy at infinity. The annihilation of the trailing images thus corresponds to the elimination of wave resistance.

On account of their perfect periodicity, the trailing images due to a source located at $(0;-y_o)$ are annihilated by the images due to a source of equal but opposite strength (a sink) located at $(n\lambda;-y_o)$, where n is any integer. This source-sink pair can represent a closed body, and this set of flows without wave resistance is thus of practical interest.

To these critically spaced source-sink pairs can be added on certain general distributions of horizontal dipoles,

representing alterations to the volume of the source-sink body, but without causing wave resistance. These dipole distributions are:

$$\mu_x(x;y_o) + \mu_x(x + \frac{n^*\lambda}{2}; y_o) \quad n^*, \text{ any odd integer} \qquad (6.1)$$

where μ_x is perfectly general. The image singularities due to $\mu(x + n^*\lambda/2)$ are precisely π radians out of phase with those due to $\mu(x)$ and thus annihilate them. The overall length of the singularity distributions so created must be greater than λ, so that

$$F = \frac{U_o^2}{gL} < \frac{1}{2\pi} \qquad (6.2)$$

and where L is the body length. However, under this condition an infinite variety of shapes can be found without wave resistance, see Figure 1. For example, distributions of the type discussed here can be superimposed in the vertical direction and also summed over the odd integer n^* for odd n^* between 1 and L/λ.

For $F > 1/2\pi$, practical shapes constructed only of sources and/or horizontal dipoles do not exist with zero wave resistance. The reason is that the body length is shorter than the wave length and constructive wave interference between different parts of the body is no longer possible.

Vortex distributions of the same form as (6.1) are also resistance free and provide net lift. They may furthermore be combined with source distributions to accomplish wave cancellation, both through constructive interference of the type involved above (critical spacing), and through combining with

source singularities at the same location so as to provide wave free flows, even for $F > 1/2\pi$. The compound singularities formed in the latter way are the main subject of the present paper, and are developed below.

7. Wave Free Compound Singularities (Planar Flow). Suppose that Ψ_h is a hard singularity system, and Ψ_s is a soft system, defined so that:

$$I[\Psi_h] = I[\Psi_h'] = 0 \qquad y = 0 \qquad (7.1)$$

and,

$$R[\Psi_s] = R[\Psi_s'] = 0 \qquad y = 0 \qquad (7.2)$$

It follows that the sum of these flows:

$$\Psi = \Psi_h + \Psi_s \qquad (7.3)$$

will satisfy the free surface boundary condition:

$$R[\Psi' + ig/U_o^2\Psi] = 0 \qquad y = 0 \qquad (3.4)$$

provided that

$$R[\Psi_h' + ig/U_o^2\Psi_s] = 0 \qquad y = 0 \qquad (7.4)$$

Since $I[\Psi_h' + ig/U_o^2\Psi_s] = 0$ in view of (7.1) and (7.2),

then,

$$\Psi_s = \frac{iU_o^2}{g}\Psi_h' \qquad (7.5)$$

and,

$$\Psi = \Psi_h + \frac{iU_o^2}{g}\Psi_h' \qquad (7.6)$$

satisfies the free surface boundary condition for each Ψ_h, and is wave free as there are no trailing images.

If, therefore,

$$2\pi\Psi_h = (m+i\Gamma)\ \ln\ (z+z_o) + (m-i\Gamma)\ \ln\ (z-z_o) \tag{7.7}$$

then,

$$2\pi\Psi_s = \left(-\frac{U_o^2\Gamma}{g} + im\frac{U_o^2}{g}\right)(z+z_o)^{-1} + \left(\frac{U_o^2\Gamma}{g} + im\frac{U_o^2}{g}\right)(z-z_o)^{-1} \tag{7.8}$$

To aid in interpreting this result, recall that:

$$2\pi\ \Psi_d = \frac{(\mu_x + i\mu_y)}{(z+z_o)} \tag{7.9}$$

where Ψ_d is the generalized potential of a point doublet.

These results, (7.7)-(7.9) permit the following interpretations:

1) Planar wave free singularities of two distinct types exist.

2) Type I. A horizontal doublet of strength μ_x located beneath a free surface, plus a vortex at the same location, of strength $\Gamma = -\mu_x g/U_o^2$ produces a wave free field; the appropriate images are simple and negative, see Figure 2. This has been earlier noted by Vladimirov [1955].

   The horizontal doublet represents flow due to displacement and the vortex a downward lift. An approximate interpretation of a concentrated compound singularity of Type I is that it represents the flow past a circular cylinder with negative circulation, such as might be produced by a suitable rotation of the cylinder about its axis, as

shown schematically in Figure 2. Notice that the lift required is independent of speed. The shape of the free surface, shown in the figure is also independent of U_o or g.

For slender bodies, wave free flows include cavity flows as a special case, as discussed in Section 10 later, and as earlier noted by Fruman [1965].

3) Type II. A source of strength m located beneath a free surface, plus a vertical doublet at the same location, of strength + mU_o^2/g produces a wave-free field; the appropriate images are simple. The net lift on a closed body created with type II singularities will be zero, but the body must possess a lift distribution along its length in proper relation to its thickness. An example is given in Figure 3. Note that the required lift distribution vanishes in the limit of low speed.

8. Three Dimensional Flows. Ship flows are symmetric about the plane $z = 0$. The potential for a submerged source is well-known, Havelock [1927]. (It has not yet been expressed in terms of image singularity distributions, if, indeed, this is possible.) The wave pattern is complex, corresponding to a spectrum of waves with directions varying over the π radian region behind the singularity. The resulting wave amplitudes decay downstream. The elimination of wave resistance through critical spacing of sources and horizontal dipoles is not possible. In fact, Krein (see Kostyukov [1959]) has shown rigorously that for a ship of finite dimensions the Michell resistance is always greater than zero; the Michell resistance is

that due to representation of the ship hull by a suitable source distribution on the plane $z = 0$. Nevertheless, wave free compound singularities do exist in three dimensions, as shown below.

9. Wave Free Compound Singularities (Three Dimensions). Consider the soft singularity system:

$$\varphi_s = \frac{\tilde{\mu}_x}{4\pi} \frac{\partial \varphi_M}{\partial x} (x-x_o;y+h;z) - \frac{\tilde{\mu}_x}{4\pi} \frac{\partial \varphi_M}{\partial x} (x-x_o;y-h;z) \tag{9.1}$$

Then $\varphi_s + \varphi_h$ will satisfy the free surface condition (3.2) provided that:

$$\frac{\partial^2 \varphi_h}{\partial x^2} = - \frac{g}{U_o^2} \frac{\tilde{\mu}_x}{2\pi} \left[\frac{\partial^2 \varphi_M}{\partial x \partial y} (x-x_o;y+h;z)\right] \qquad y = 0$$

or,

$$\frac{\partial \varphi_h}{\partial x} = - \frac{g}{U_o^2} \frac{\tilde{\mu}_x}{2\pi} \left[\frac{\partial \varphi_M}{\partial y} (x-x_o;y+h;z)\right] \qquad y = 0 \tag{9.2}$$

But

$$\frac{\partial \varphi_h}{\partial x} = - \frac{\partial \varphi_h}{\partial x_o}, \text{ so:}$$

$$\frac{\partial \varphi_h}{\partial x_o} = + \frac{g}{U_o^2} \frac{\tilde{\mu}_x}{2\pi} \left[\frac{\partial \varphi_M}{\partial y} (x-x_o;y+h;z)\right] \qquad y = 0 \tag{9.3}$$

The appropriate hard system is thus,

$$\varphi_h = \frac{g}{U_o^2} \frac{\tilde{\mu}_x}{4\pi} \left[\int_{x_o}^{\infty} \frac{\partial \varphi_M}{\partial y} (x-x'';y+h;z)dx'' - \int_{x_o}^{\infty} \frac{\partial \varphi_M}{\partial y} (x-x'';y-h;z)dx''\right] \tag{9.4}$$

representing a uniform distribution of vertical doublets of strength $(g/U_o^2)\bar{\mu}_x$ trailing behind the horizontal doublet at $(x_o;-h;0)$, plus its negative image. This distribution of vertical doublets corresponds exactly to the vortex of strength Γ in the two-dimensional case, Type I of Section 7. Again, for slender bodies, wave free flows include cavity flows as a special case.

The three dimensional wave free flow corresponding to Type II for planar flows involves an infinite distribution of singularities extending vertically downward. They may be derived as follows. If the hard system is,

$$\varphi_h = \frac{M}{4\pi}\,\varphi_M(x-x_o;y+h;z) + \frac{M}{4\pi}\,\varphi_M(x-x_o;y-h;z) \qquad (9.5)$$

then the soft potential must satisfy on the free surface,

$$\frac{\partial\varphi_s}{\partial y} = -\frac{U_o^2}{g}\,\frac{M}{2\pi}\left[\frac{\partial^2\varphi_M}{\partial x^2}(x-x_o;y-h;z)\right] \qquad y = 0 \qquad (9.6)$$

or since

$$\frac{\partial^2\varphi_M}{\partial x^2} = -\left(\frac{\partial^2\varphi_M}{\partial y^2} + \frac{\partial^2\varphi_M}{\partial z^2}\right),$$

$$\varphi_s = \frac{U_o^2}{g}\,\frac{M}{2\pi}\,\frac{\partial\varphi_M}{\partial y}(x-x_o;y-h;z) + \frac{U_o^2 M}{g\cdot 2\pi}\int_h^\infty \frac{\partial^2\varphi_M}{\partial z^2}(x-x_o;y-y';z)\,dy' \qquad y = 0 \qquad (9.7)$$

The appropriate soft system is thus:

$$\varphi_s = \frac{U_o^2}{g}\frac{M}{4\pi}\left[\frac{\partial\varphi_M}{\partial y}(x-x_o;y+h;z) + \frac{\partial\varphi_M}{\partial y}(x-x_o;y-h;z)\right]$$

$$+ \frac{U_o^2}{g}\frac{M}{4\pi}\left[\int_h^{\infty}\frac{\partial^2\varphi_M}{\partial z^2}(x-x_o;y-y';z)dy' - \int_{-h}^{-\infty}\frac{\partial^2\varphi_M}{\partial z^2}(x-x_o;y-y';z)dy'\right] \tag{9.8}$$

The first term in this soft potential represents a vertical doublet of strength U_o^2M/g located at the position of the source, plus its positive image; this term corresponds exactly to the planar doublet of Type II. In addition, however, a vertical distribution of quadrupoles extending from the source to infinity in the vertical direction is required, plus its negative image. This wave free flow is shown schematically in Figure 4.

It can readily be shown that the free surface boundary condition (3.2) is satisfied on the track of the ship ($y = 0$; $z = 0$) by:

$$\varphi = \frac{M}{4\pi}\left\{\varphi_M(x-x_o;\ y;\ z) + \varphi_M(x-x_o;\ y;\ z) + \frac{2U_o^2}{g}\left(\frac{\partial\varphi_M}{\partial y}(x-x_o;\ y;\ z) + \frac{\partial\varphi_M}{\partial y}(x-x_o;\ y;\ z)\right)\right\} \tag{9.9}$$

eliminating the infinite distribution of quadrupoles in (9.8). Although (9.9) does not represent a wave-free singularity, it is of interest in view of its approximation to the free surface boundary conditions near the ship, implying small waves elsewhere.

Applications and Interpretations

10. Buoyancy Cancellation Cavity Flows. When the submerged body corresponding to the singularity distribution is sufficiently slender, simple physical interpretations of the singularities are possible. A horizontal dipole distribution corresponds to a distribution of cross-sectional area, $A(x)$, or thickness, $t(x)$:

$$\tilde{\mu}_x(x) = U_o A \; ; \; M = U_o \frac{dA}{dx}$$
$$\mu_x(x) = U_o t \; ; \; m = U_o \frac{dt}{dx} \tag{10.1}$$

The vertical dipole distribution corresponds to a distribution of vertical force, ℓ; defined so that $\ell(x)$ is the net vertical force acting on the body between $(0,x)$:

$$\frac{d\tilde{\ell}}{dx} = -\rho U_o \frac{d\tilde{\mu}_y}{dx} \tag{10.2}$$

$$\frac{d\ell}{dx} = +\rho U_o \Gamma = -\rho U_o \frac{d\mu_y}{dx} \tag{10.3}$$

In the case of singularity systems of Type I, (9.4) is equivalent to:

$$\frac{d\tilde{\mu}_y}{dx} = \frac{g}{U_o^2} \tilde{\mu}_x \tag{10.4}$$

and the same relation holds in planar flow.

Combining (10.1)-(10.4), leads to a relation between the distribution of vertical dynamic and buoyancy force:

$$\frac{dL}{dx} = -\rho g A \; ; \; \frac{d\ell}{dx} = -\rho g t \qquad (10.5)$$

These relations show that at each section of a body corresponding to wave free singularities of Type I, dynamic lift (downward) is generated exactly to annul the upward buoyancy force. In fact, it may also be shown that in three dimensions the dynamic pressures are so distributed around the circumference of a cross-section as to balance the hydrostatic gradient at each point thereon. The pressure distribution on such a body is therefore completely analogous to that on a body of revolution in isolated flow, neglecting gravity.

Distributions of Type I can be used to generate an infinite variety of closed forms, including submarine-like shapes, which have no wave drag at any specified Froude number. This result is contrary to the general belief that three dimensional bodies without wave resistance do not exist. Of course, these bodies are not practical in naval architecture since they have no support capability.

In a cavity flow, the shape of the long trailing cavity (in which the pressure is constant) must adjust itself so that the buoyancy at each section is exactly balanced by dynamic lift. Therefore, a cavity must be represented by singularities of Type I. As a result, cavities created beneath a free surface are not of themselves a source of trailing waves or of wave resistance. Such flows have not yet been calculated, although the planar cavity flow, deeply submerged, but including the gravity effect on the cavity, has been treated, Tulin [1965].

11. <u>Type II and Related Systems</u>. In the case of these systems, (9.9) is equivalent to:

$$\bar{\mu}_y = \frac{U_o^2 M}{g}$$
$$\mu_y = \frac{U_o^2 m}{g} \tag{11.1}$$

which provides a relation between lift distributions and body shape, in view of (10.1-10.3):

$$\frac{d\bar{\ell}}{dx} = -\rho \frac{U_o^4}{g} \frac{d^2A}{dx^2}$$
$$\frac{d\ell}{dx} = -\rho \frac{U_o^4}{g} \frac{d^2t}{dx^2} \tag{11.2}$$

Care must be taken in evaluating $\bar{\ell}$ at the ends.

What is the shape of bodies corresponding to (11.1)? A simplified answer can be obtained by visualizing the doublet $\bar{\mu}_y$ as a source-sink pair with vertical separation 2h, where $\bar{\mu}_y = M_2 \cdot 2h$. If the ship's hull is composed of a source distribution M_1 spread out around the waterline and a source distribution $M_2 = (U_o^2/gh)M_1$ spread out around the depth h, then a ship's hull with an underwater bulge appears. The bulge is mirrored in the free surface by its negative image, which produces a curvature of the flow which must be followed by the lower bulge. The ratio of the cross-sectional area of the bulge M_2 and the hull M_1 is $2U_o^2/gh$. Therefore, if $U_o^2/gh \approx 1$, then 2/3 of the forebody should be in the bulb, and 1/3 in the bow. The situation is schematically illustrated in Figure 4. Perhaps rules for the vertical distribution of cross-sectional area, based on vertical lift requirements (11.1), can be useful in ship design.

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FIGURE 1 - WAVE FREE BODIES; CRITICAL SPACING, PLANAR FLOW

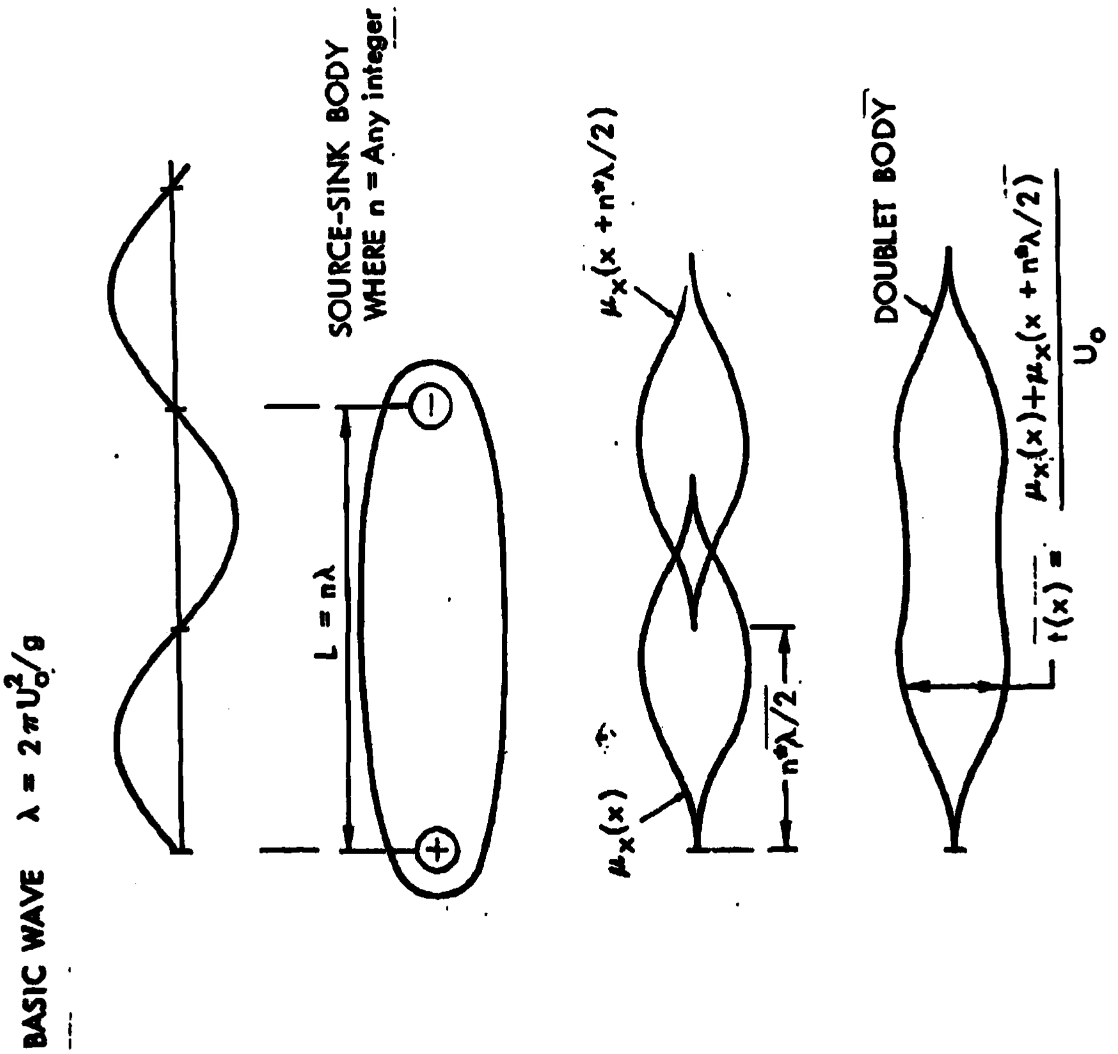


FIGURE 2 - WAVE FREE FLOWS OF TYPE I

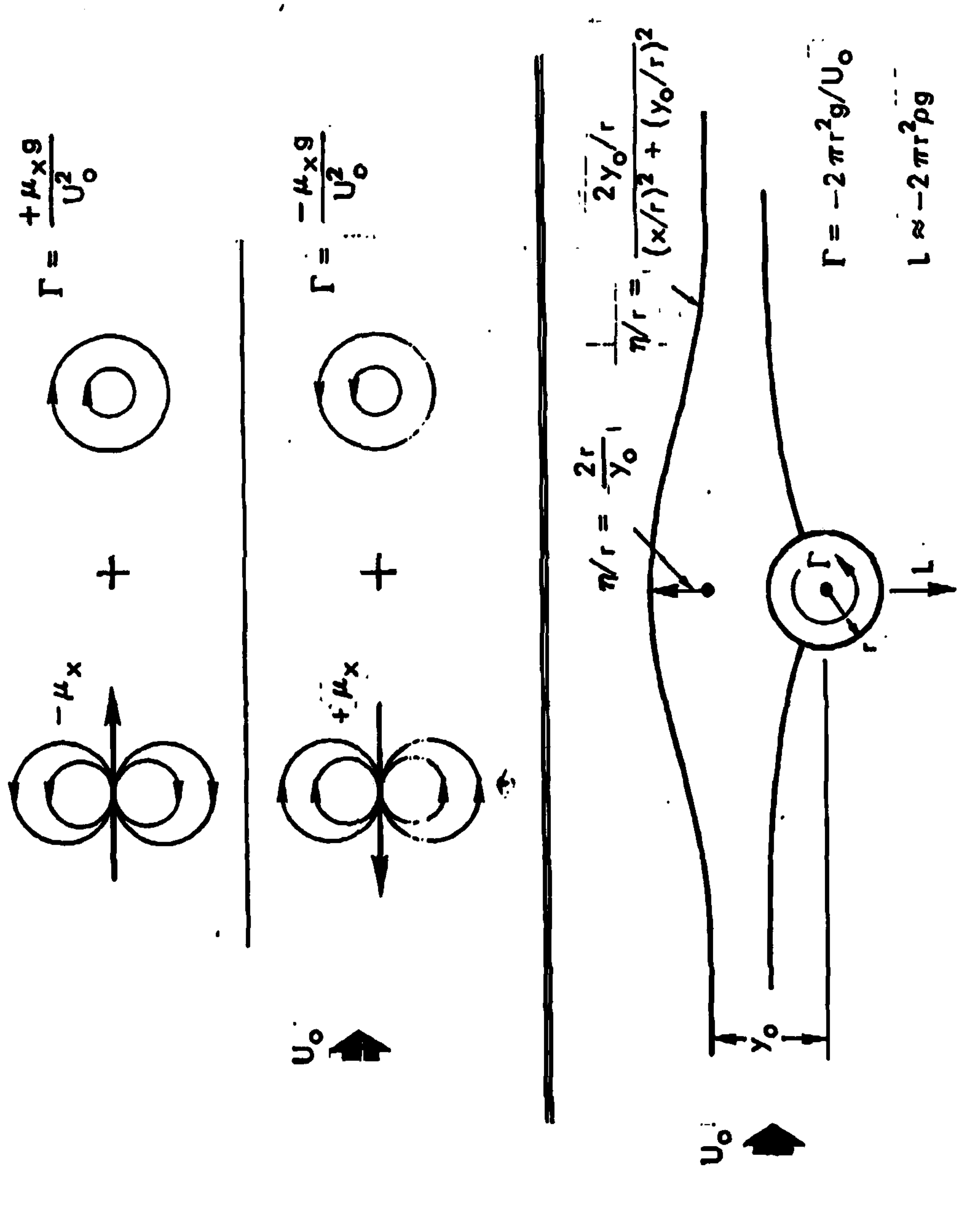


FIGURE 3 - WAVE FREE FLOWS OF TYPE II

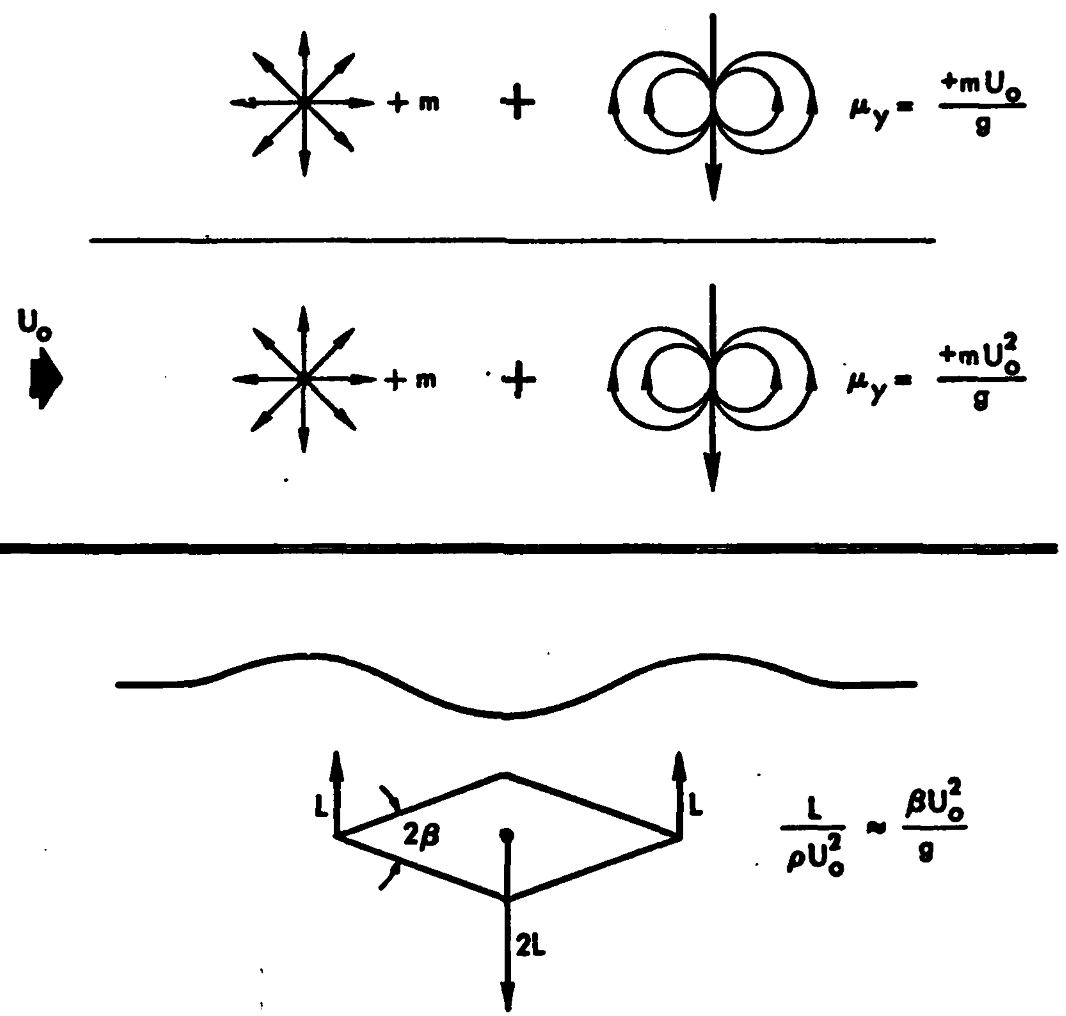
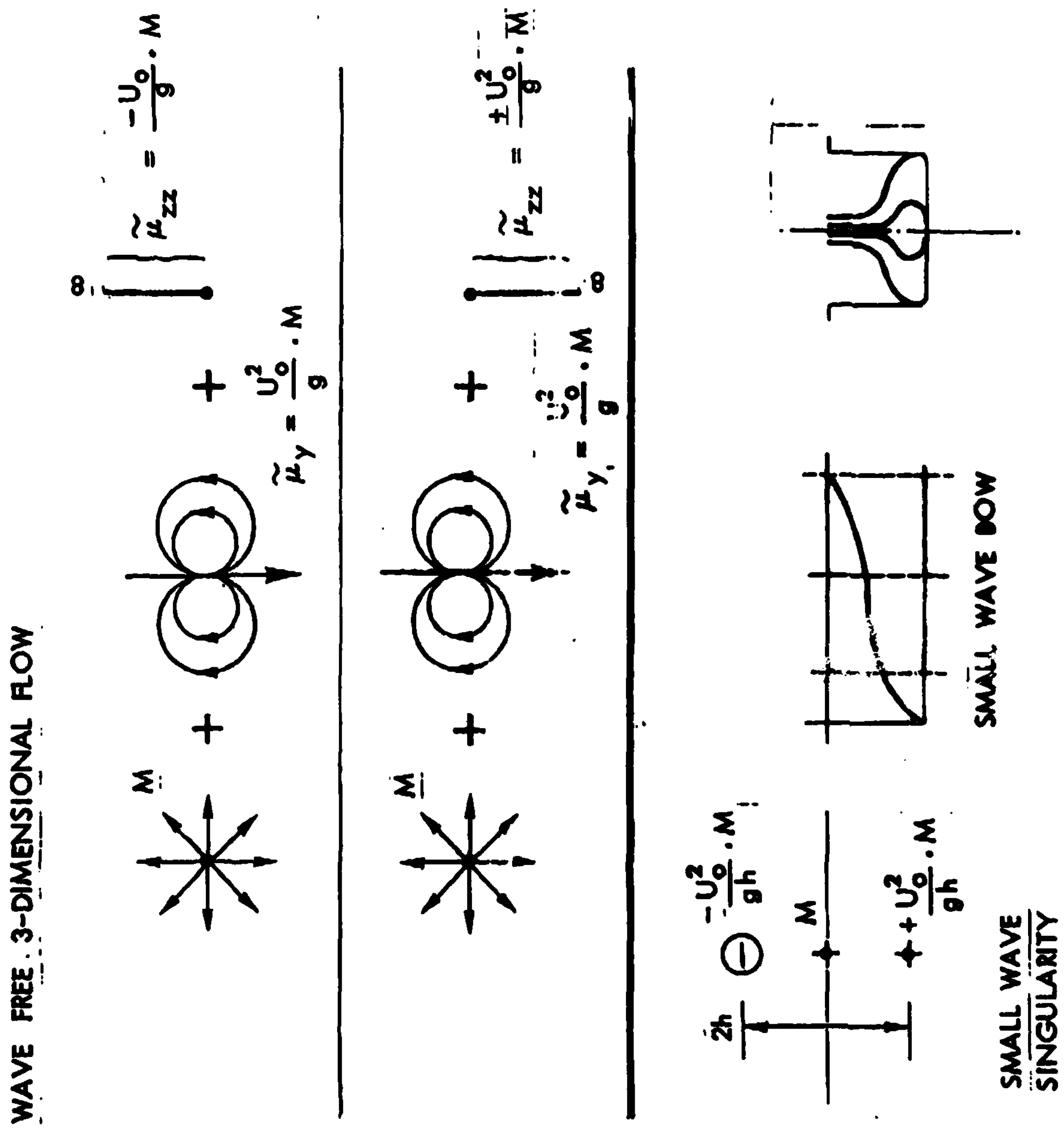


FIGURE 4 - TYPE II SINGULARITIES IN 3-DIMENSIONS



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